Qualitative Studies with Microwaves

Physics 401, Fall 2019
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AT URBANA-CHAMPAIGN

Qualitative Studies with Microwaves

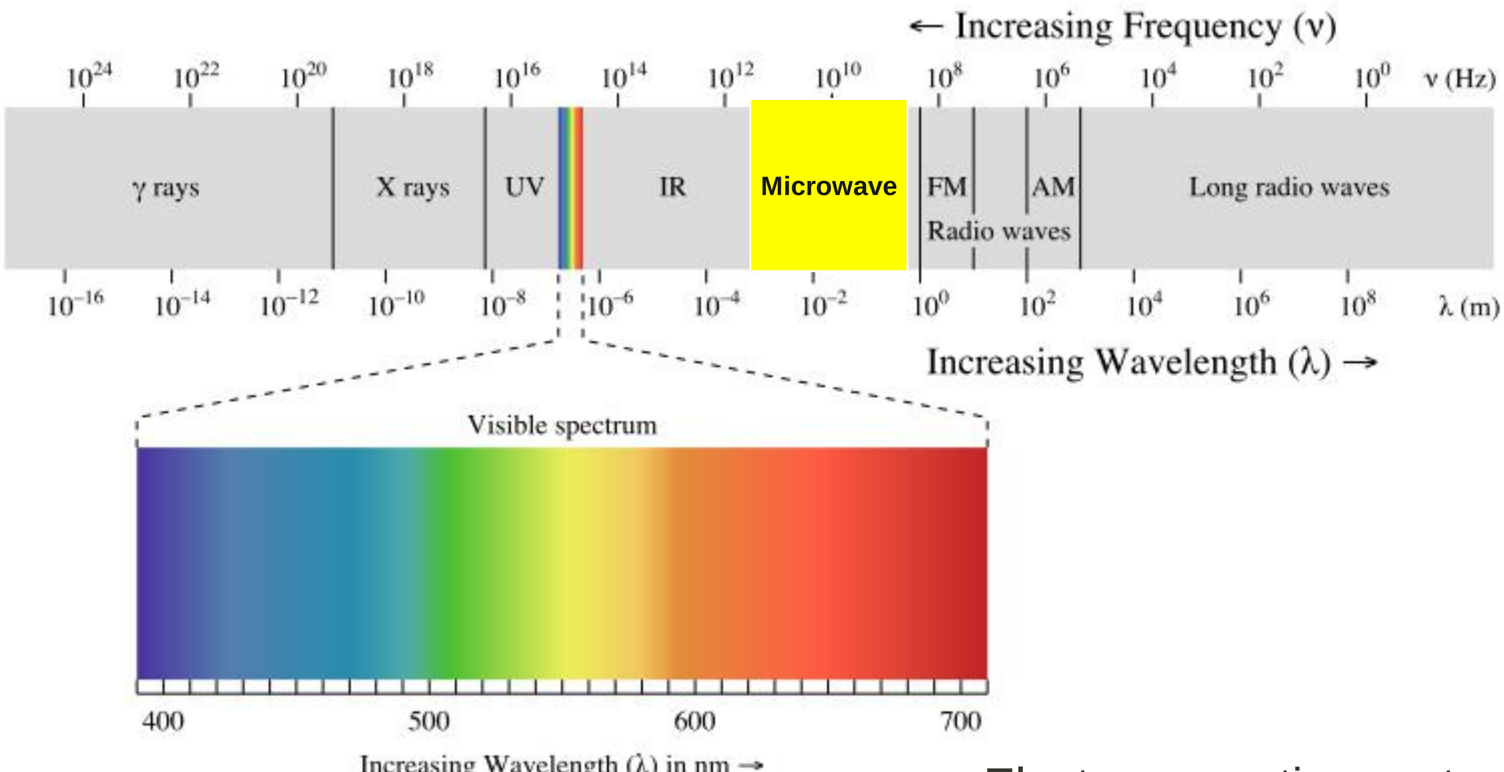
The main goals of the Lab:

- ✓ **Refreshing the memory about the electromagnetic waves propagation**
- ✓ **Microwaves. Generating and detecting of the microwaves**
- ✓ **Microwaves optics experiments**

This is two weeks Lab

Microwaves place in the electromagnetic spectrum

The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.



Electromagnetic spectrum*

Application of the microwaves

Microwave oven (2.45GHz)

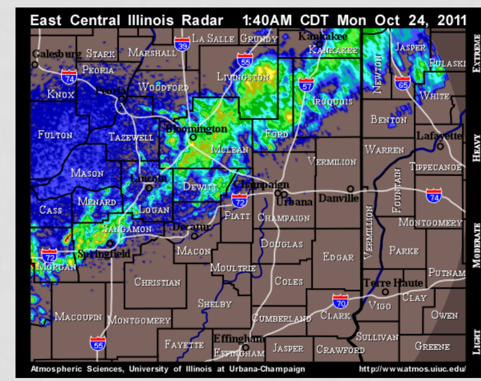
Communication (0.8-2.69GHz)

Satellite TV (4-18GHz)

Radar (up to 110GHz)

Motion detector (10.4GHz)

GPS 1.17-1.575 GHz



Weather radar (8-12Ghz)

*by courtesy Wikipedia

Maxwell equations

$$\nabla \vec{D} = \rho \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \vec{B} = 0 \quad (2)$$

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

James Clerk Maxwell
(1831–1879)

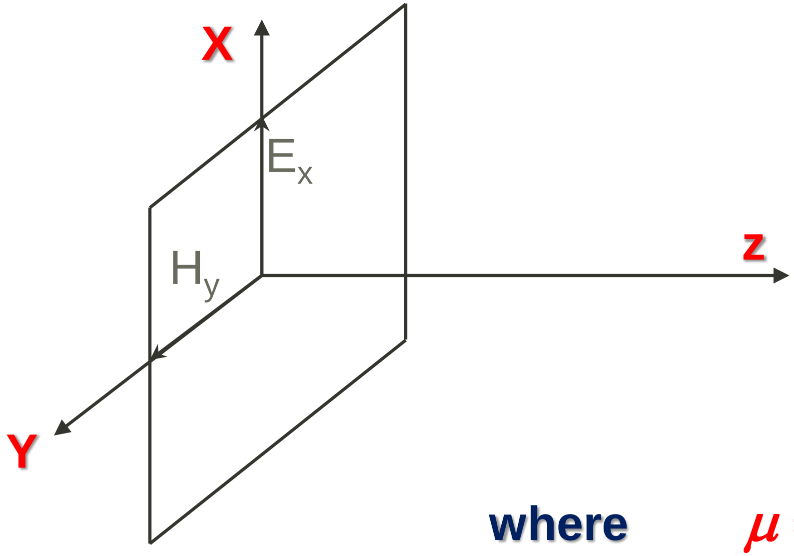
If $\rho = 0$ and $J = 0$ and taking in account that $\vec{D} = \varepsilon \vec{E}$
$\vec{B} = \mu \vec{H}$ (1) and (4) can be rewritten as

$$\nabla \vec{D} = \varepsilon \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Plane wave

Now assuming that plane wave propagate in z direction and what leads to E_y=E_z=0 and H_x=H_z=0



Now (3) and (4) could be simplified as

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \qquad (5)$$

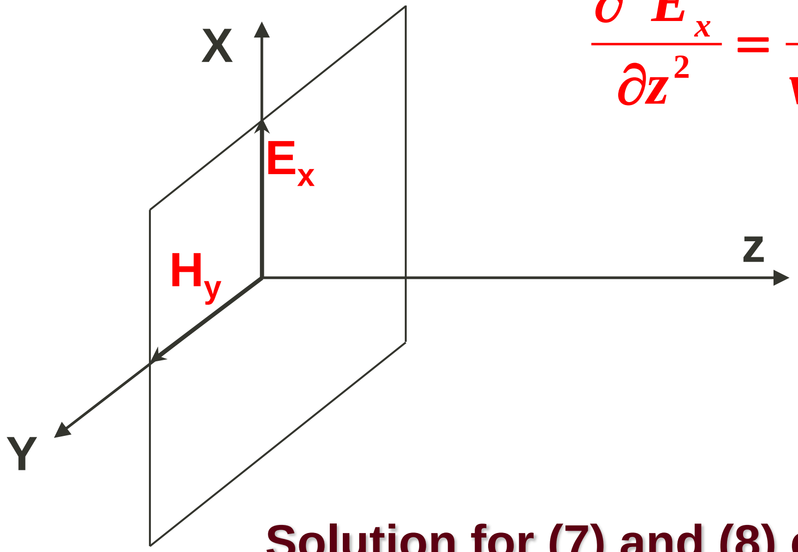
$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_y}{\partial t} \qquad (6)$$

where $\mu = \mu_o \mu_r$ $\varepsilon = \varepsilon_o \varepsilon_r$

μ_0 is the free space permeability, ε_0 is the free space permittivity
μ_r is permeability of a specific medium , ε_r is permittivity of a specific medium

Plane wave

Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:



$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \quad (7) \qquad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 H_y}{\partial t^2} \quad (8)$$

where $v = \frac{1}{\sqrt{\varepsilon\mu}}$

$$E_x = E_{x0}\cos(\omega t - kx)$$
$$H_y = H_{y0}\cos(\omega t - kx)$$

Solution for (7) and (8) can found as $H_y = \sqrt{\frac{\varepsilon}{\mu}}E_x$ or $E_x = ZH_y$

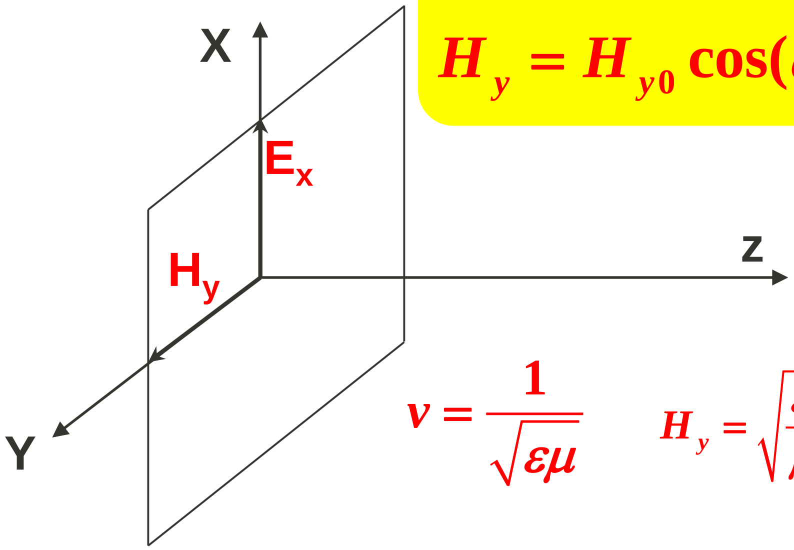
where $Z = \sqrt{\frac{\mu}{\varepsilon}}$ known as characteristic impedance of medium

k is wave vector and is defined as $k = \frac{2\pi}{\lambda}$ or $k = \frac{\omega}{v}$

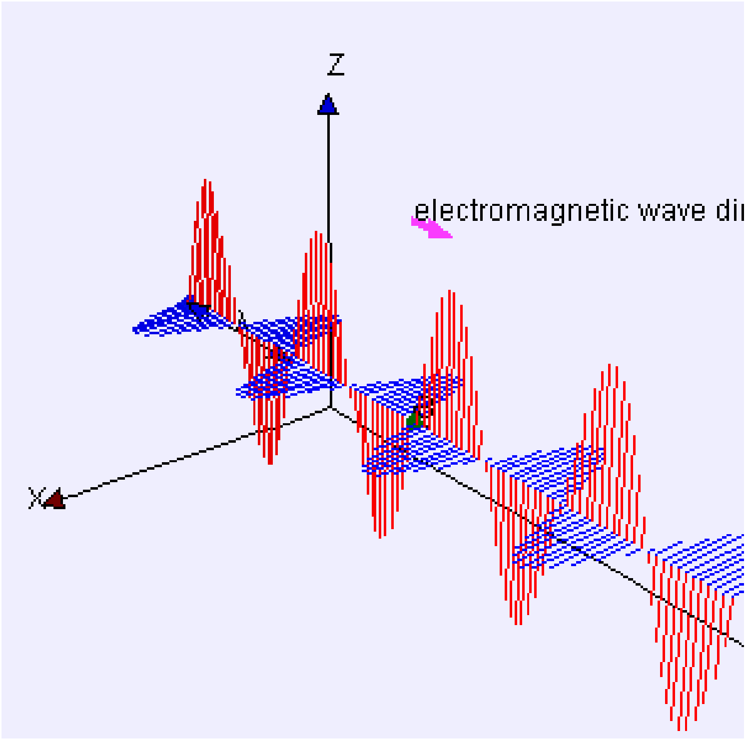
For free space ($\varepsilon_r=1$ and $\mu_r=1$) $Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 ohms$

Plane wave

$$E_x = E_{x0}\cos(\omega t - kx)$$
$$H_y = H_{y0}\cos(\omega t - kx)$$



$$v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad H_y = \sqrt{\frac{\varepsilon}{\mu}}E_x$$



$$Z = \sqrt{\frac{\mu}{\varepsilon}} \qquad E_x = ZH_y \qquad k = \frac{2\pi}{\lambda} \text{ or } k = \frac{\omega}{v}$$

$$Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377ohms$$

For free space (ε_r=1 and μ_r=1)

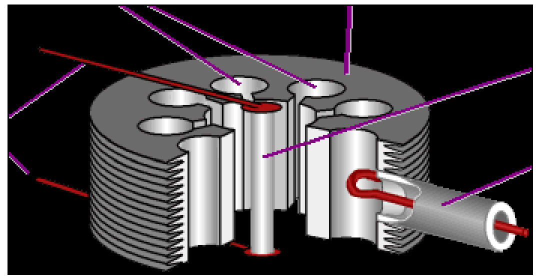
*by courtesy Wikipedia

Generating of the microwaves

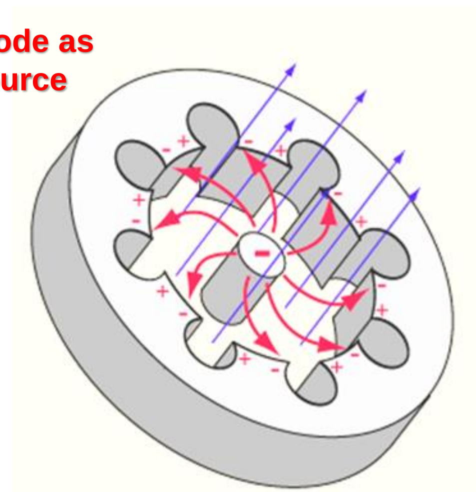
Vacuum tubes: klystron, magnetron, traveling wave tube

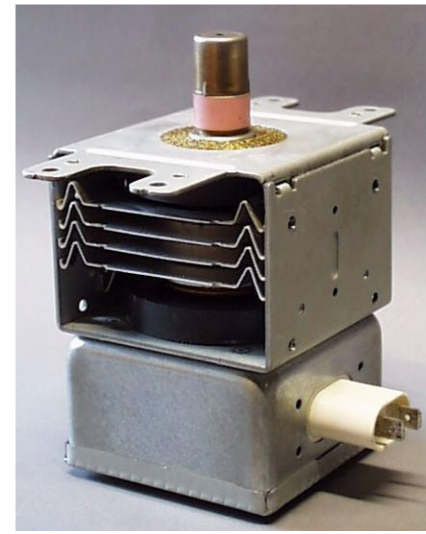
Solid state devices: FET, tunneling diodes, Gunn diodes

Tunable frequency from 9 to 10GHz; maximum output power 20mW



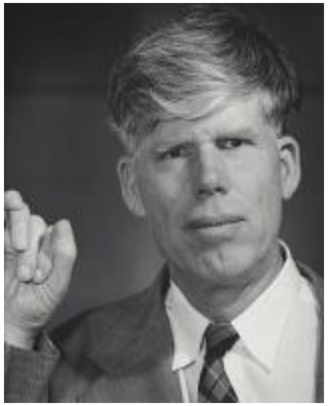
Heated cathode as electron source



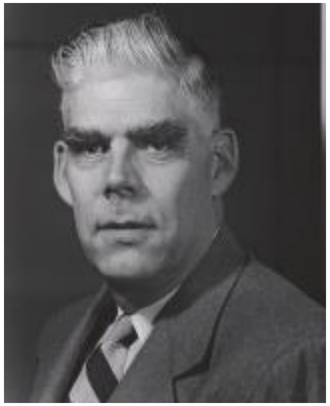


Microwave oven magnetron; typical power 0.7-1.5kW

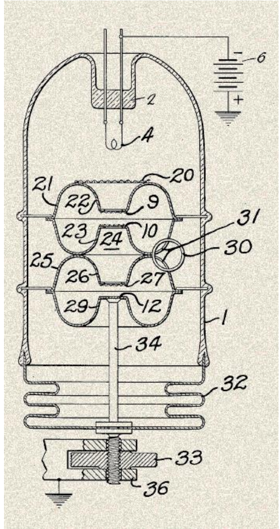
Klystron. A piece of history.



Russell Harrison Varian (April 24, 1898 – July 28, 1959)



Sigurd Fergus Varian (May 4, 1901 – October 18, 1961)



Patented May 20, 1941

2,242,275

UNITED STATES PATENT OFFICE

2,242,275

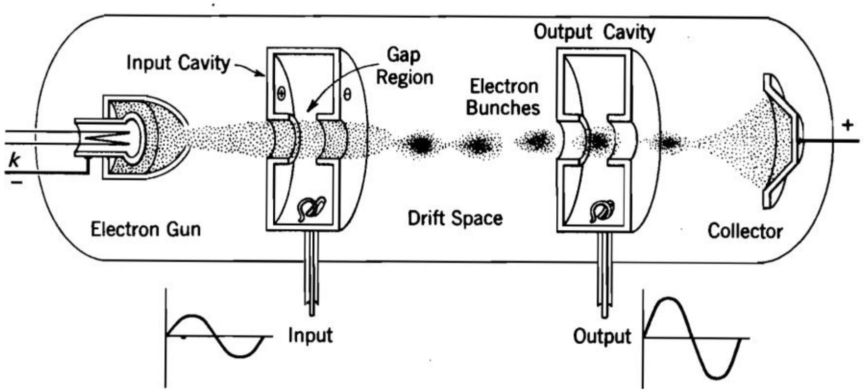
ELECTRICAL TRANSLATING SYSTEM AND METHOD

Russell H. Varian, Stanford University, Calif., assignor to The Board of Trustees of The Leland Stanford Junior University, Stanford University, Calif., a corporation of California

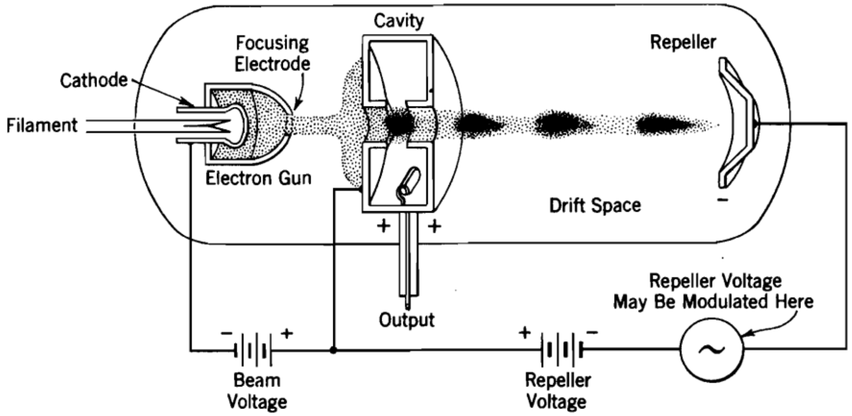
Application October 11, 1937, Serial No. 168,355

Varian Brothers...Klystron Tube (1940)

Generating of the microwaves. Klystron.



Single transit klystron



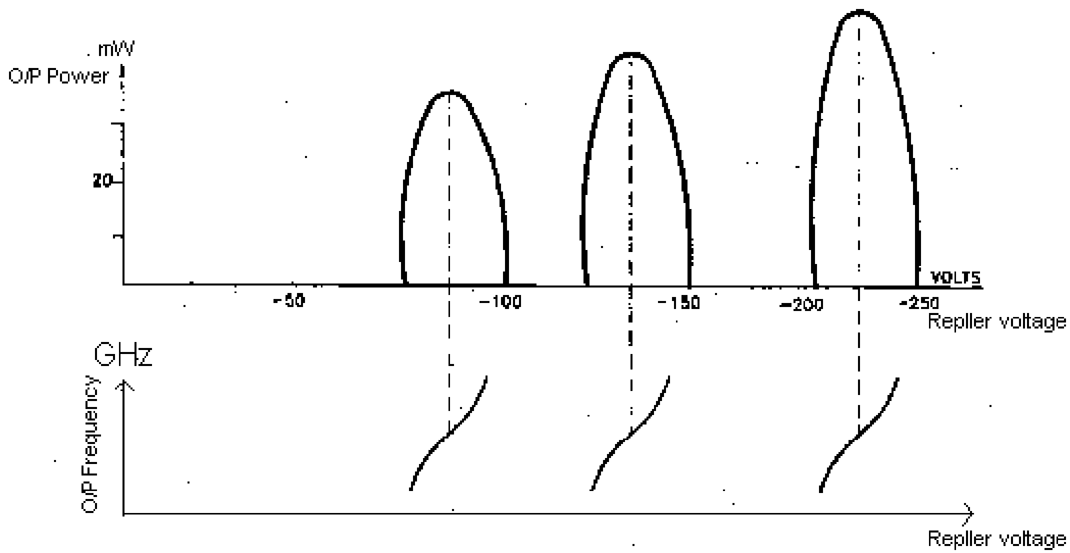
Reflection klystron

Advantages: well defined frequencies, high power output

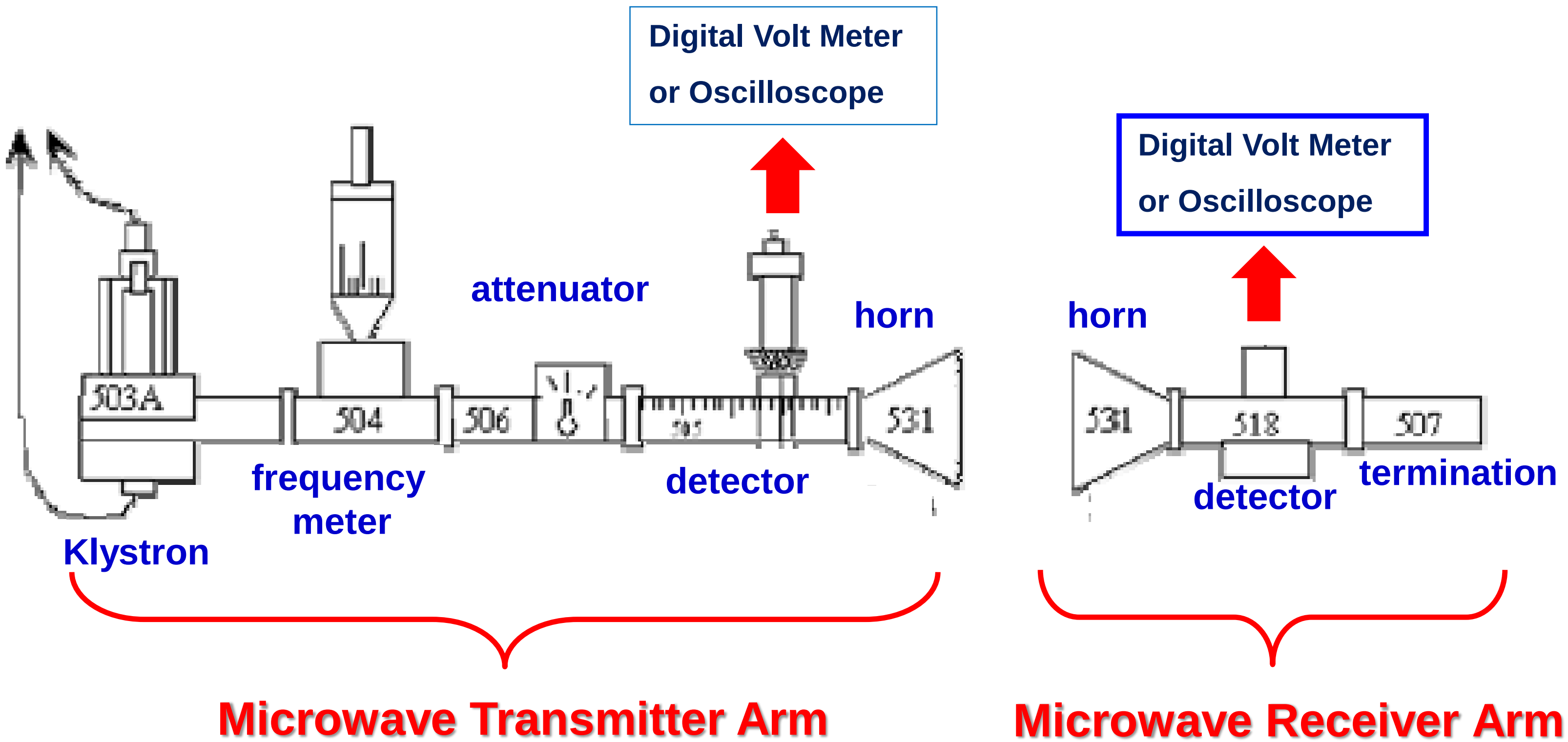
High power klystron used in Canberra Deep Space Communications Complex (courtesy of Wikipedia)

2K25 Klystron

GENERAL CHARACTERISTICS
Frequency Range ······················8,500 to 9,660 Mc
Cathode Oxide-coated, indirectly heated
Heater Voltage································6.3Volts
Heater Current·························0.44 Amperes

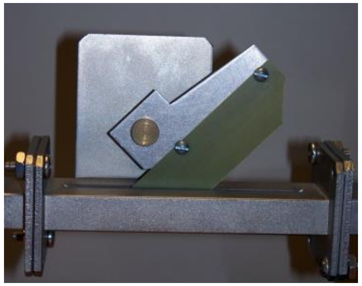


Experimental setup. Main components.



Experimental setup. Main components.

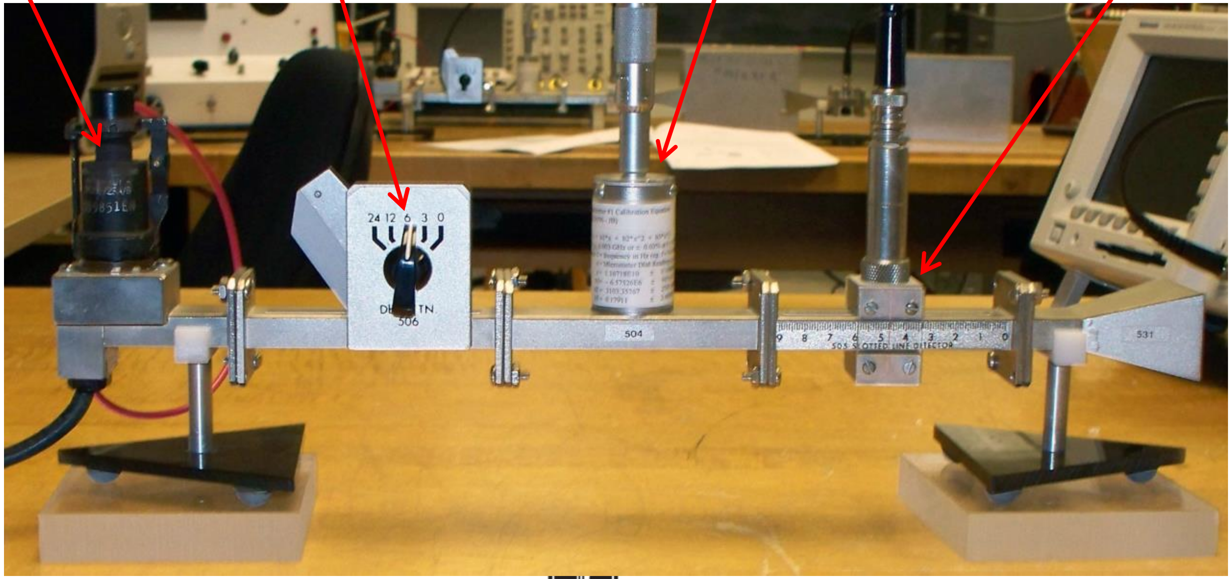
Klystron



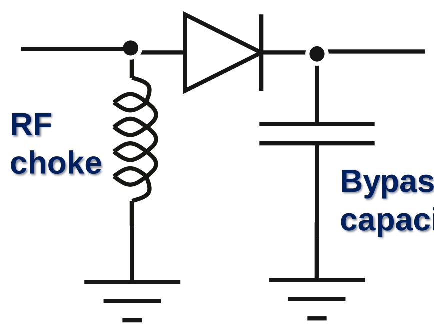
Attenuator

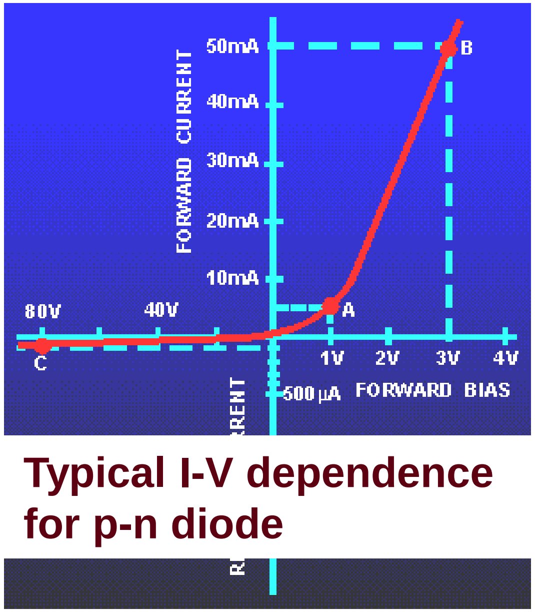
Frequency meter

detector



Detecting of the microwaves





Typical I-V dependence for p-n diode

$$I = I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Taylor expansion for exp function will give

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$I \propto aV + bV^2 + \ldots$$

0(DC)

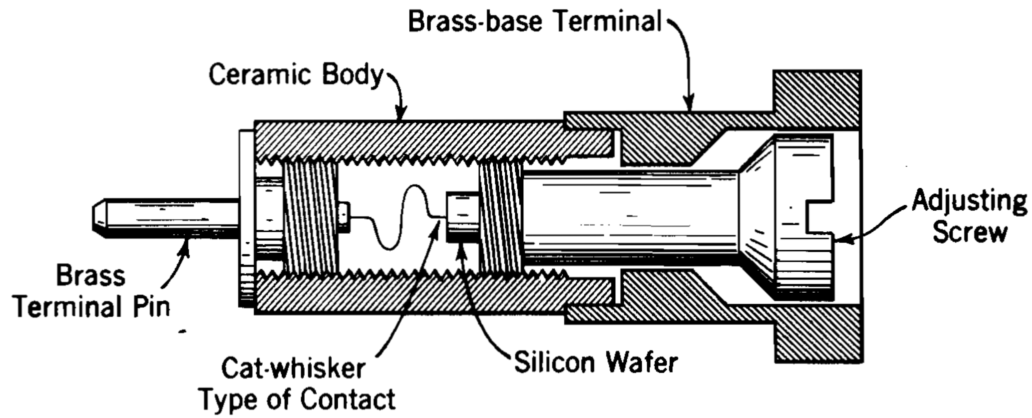
If V=V$_0$sinωt

$$b * \frac{V^2{}_0}{2}(1 - \cos 2\omega t)$$

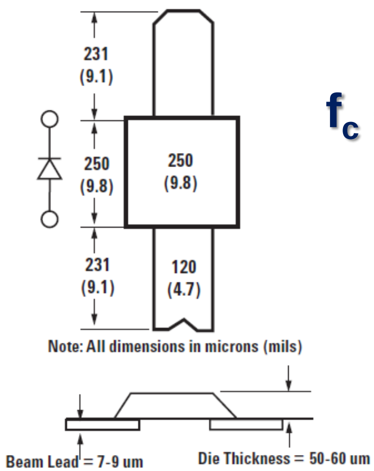
And finally

$$I_{DC} \propto b\frac{V_0{}^2}{2} + \ldots$$

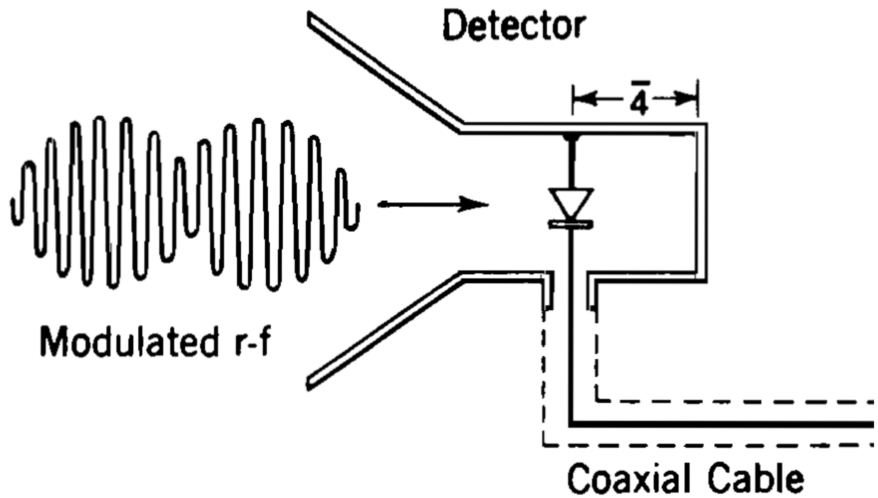
Detecting of the microwaves



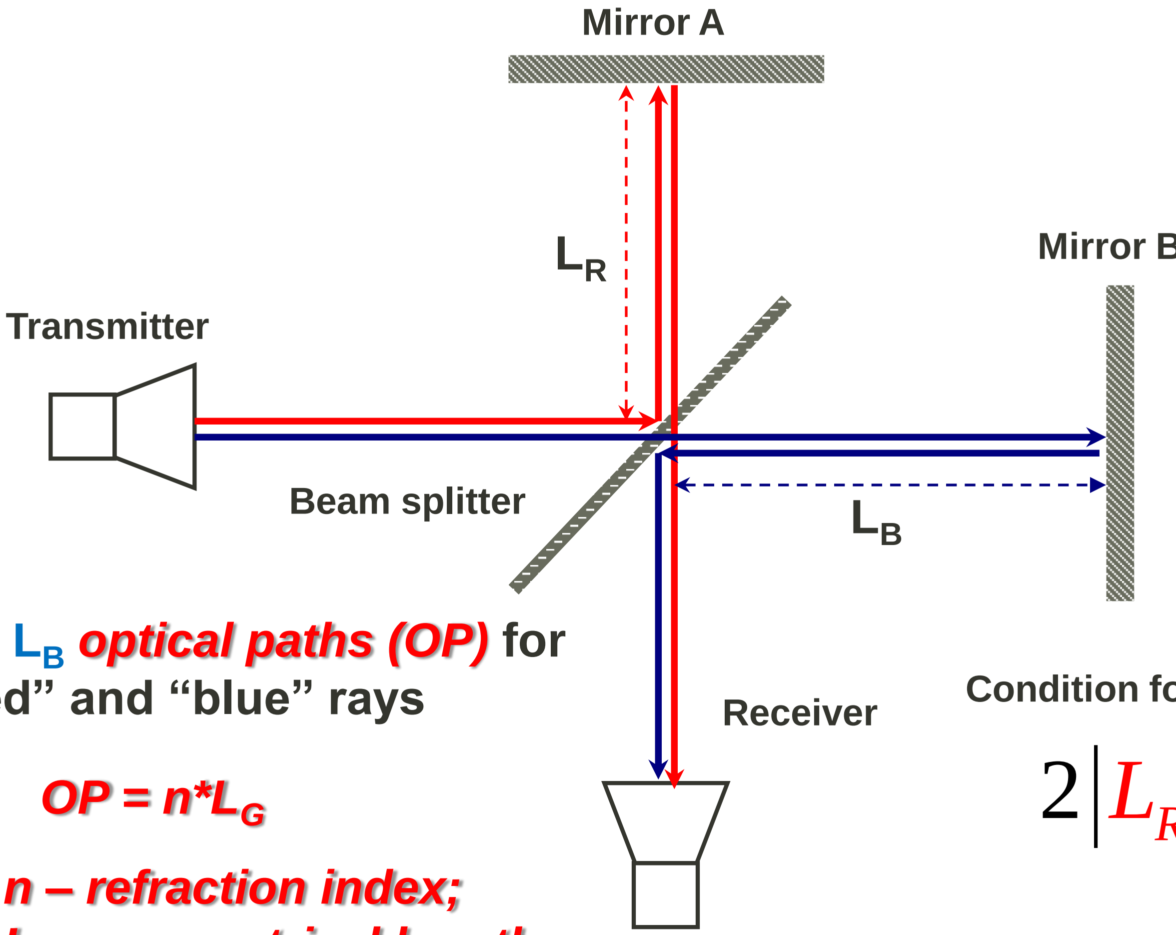
HSCH-9161
HSCH-9162
GaAs Detector Diode

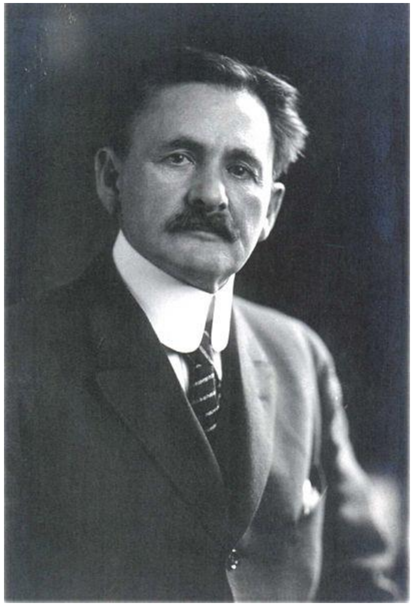


f_c ~ 200GHz



Experiments: Michelson interferometer





Albert Abraham Michelson

(1852 - 1931)

The Nobel Prize in Physics 1907

L_R, L_B ***optical paths (OP)*** **for "red" and "blue" rays**

OP = n*L_G

$$OP = n^*L_G$$

n – refraction index;

L_G – geometrical length

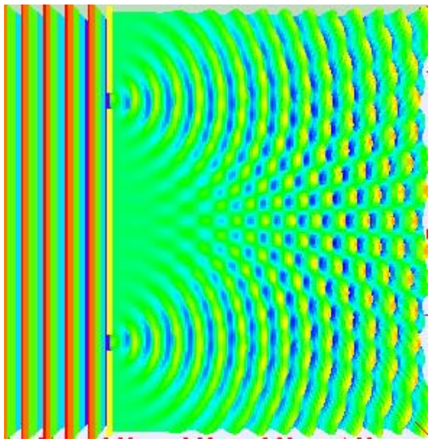
Condition for constructive interference

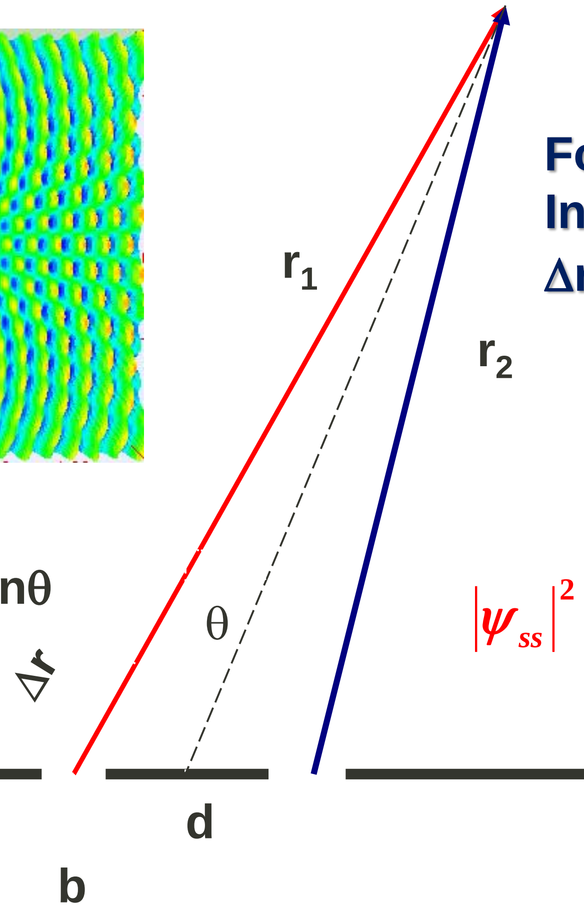
$$2\left|L_R - L_B\right| = k\lambda$$

Experiments:
Michelson interferometer

Physics 403 Lab Michelson interferometer setup

Experiments: Double slit Interference. T. Young 1801





$\Delta r = r_1 - r_2 = d\sin\theta$

For constructive Interference
$\Delta r = n\lambda$ or $d\sin\theta = n\lambda$

Thomas Young
(1773 – 1829)

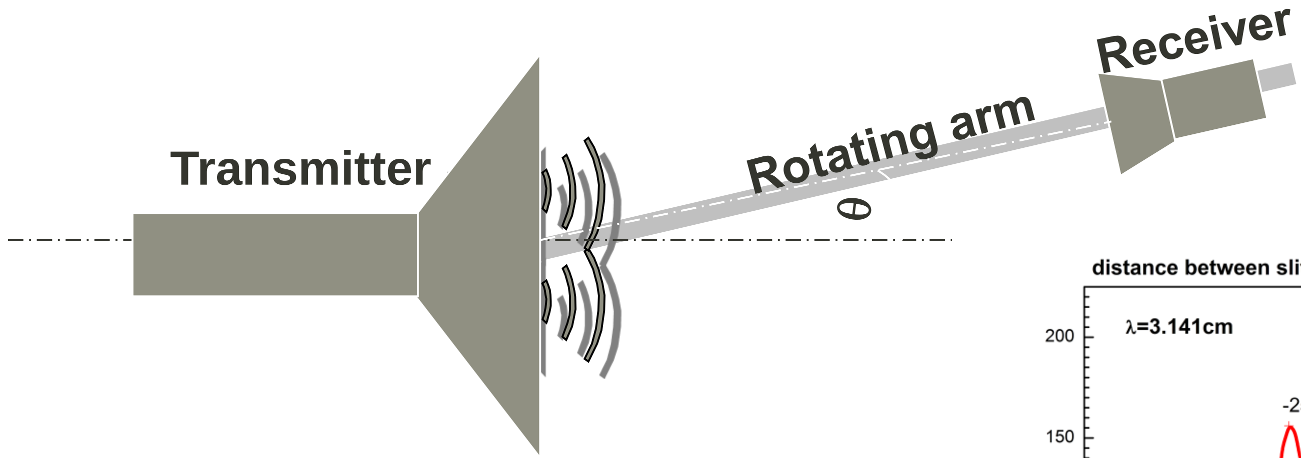
The measured envelope of the diffraction pattern can be defined as:

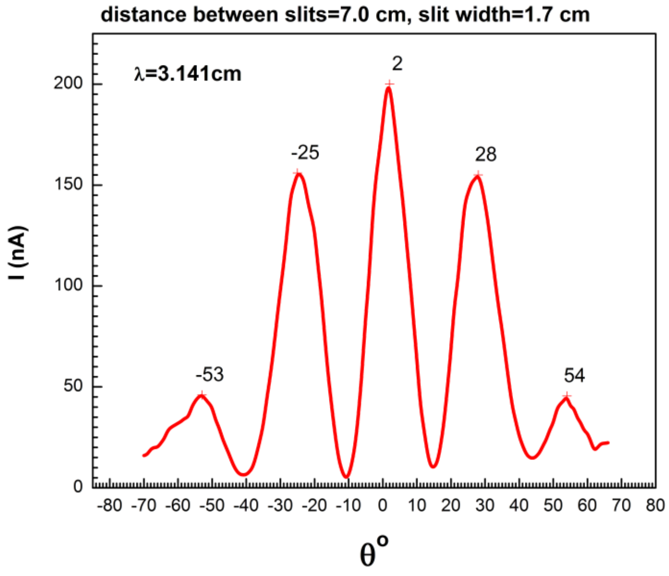
$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \times \cos^2\left[(kd\sin(\theta/2)\right]$$

where $x = kb\sin(\theta/2)$ and

$k = \frac{2\pi}{\lambda}$ is wave vector of the plane wave

Experiments: Double slit interference



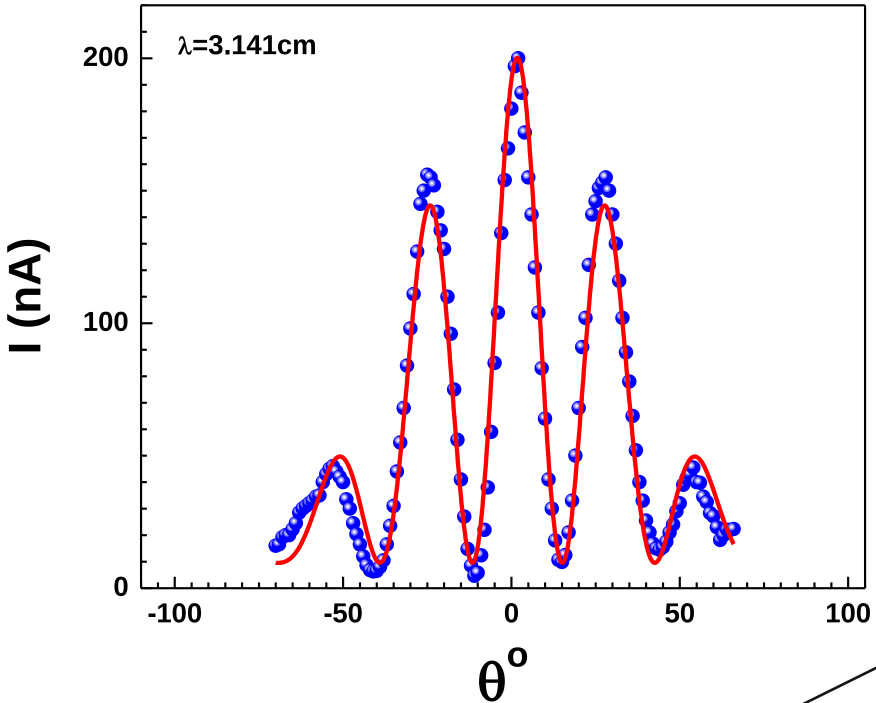


Physics 401 Lab setup and example of the data

Experiments: Double slit interference. Fitting

$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \times \cos^2[(kd\sin(\theta/2)]$$

$$x = kb\sin(\theta/2)$$



Model	Two_slit (User)		
Equation	y=I0*(sin(K1*sin(pi*x/360+f))/(K1*sin(pi*x/360+f)))^2 *(cos(K2*sin(pi*x/360+f)))^2+I00		
Reduced Chi-Sqr	94.62111		
Adj. R-Square	0.96659	Value	Standard Error
	I0	190.6014	3.042882
	K1	4.384042	0.074754
	K2	13.51332	0.052244
	f	-0.01525	7.19E-04
	I00	9.572049	1.440409

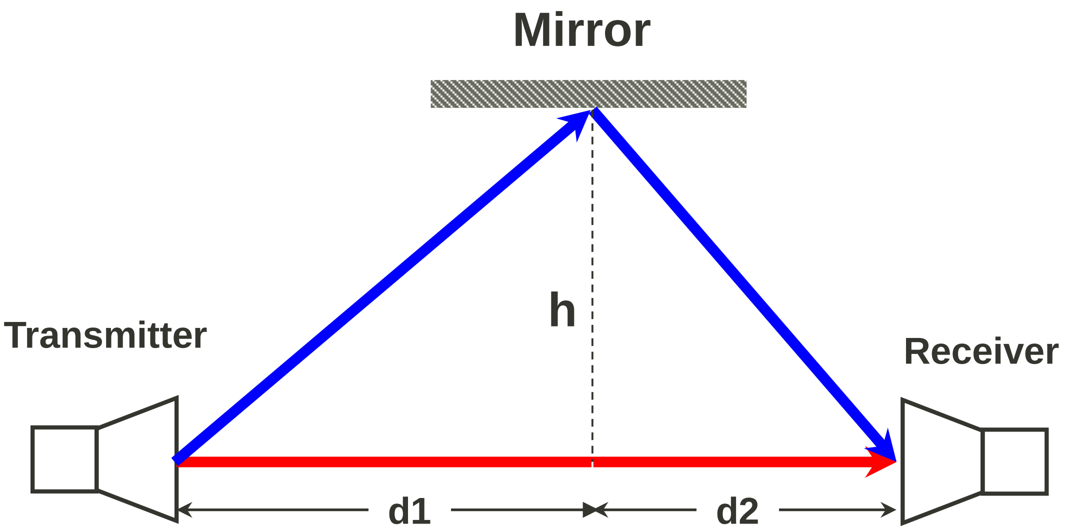
Fitting equation

$$y = I0 \cdot \left(\frac{\sin\left(K1\sin\left(\frac{\pi x}{360}+f\right)\right)}{K1\sin\left(\frac{\pi x}{360}+f\right)}\right)^2 \cos^2\left(K2\sin\left(\frac{\pi x}{360}+f\right)\right) + I00$$

Here in fitting expression:

$$I_0 = |\psi_0|^2;$$
$$K1 = kb;$$
$$K2 = kd$$

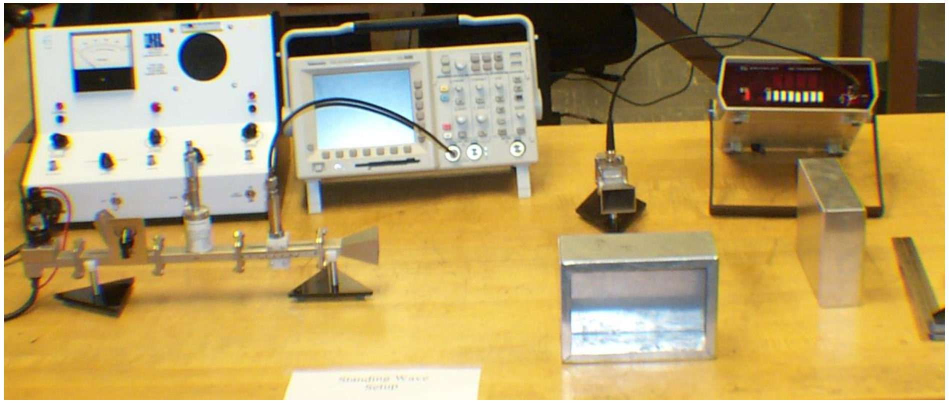
Lloyd's Mirror experiment



Humphry Lloyd
1802-1881

Difference of the wave paths of "red" and "blue" rays is:

$$\Delta S = \sqrt{h^2 + d1^2} + \sqrt{h^2 + d2^2} - (d1 + d2)$$



Lab setup picture

For constructive interference

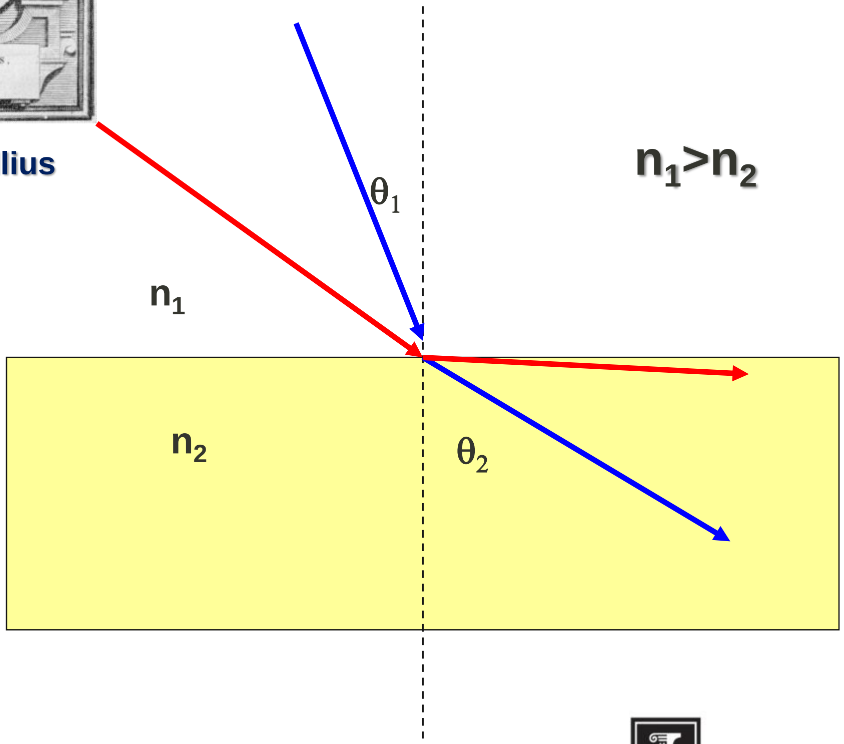
$$\Delta S = n\lambda$$

Total internal reflection experiment. Snell's law

Willebrord Snellius
1580-1626

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Snell's law



$n_1 > n_2$

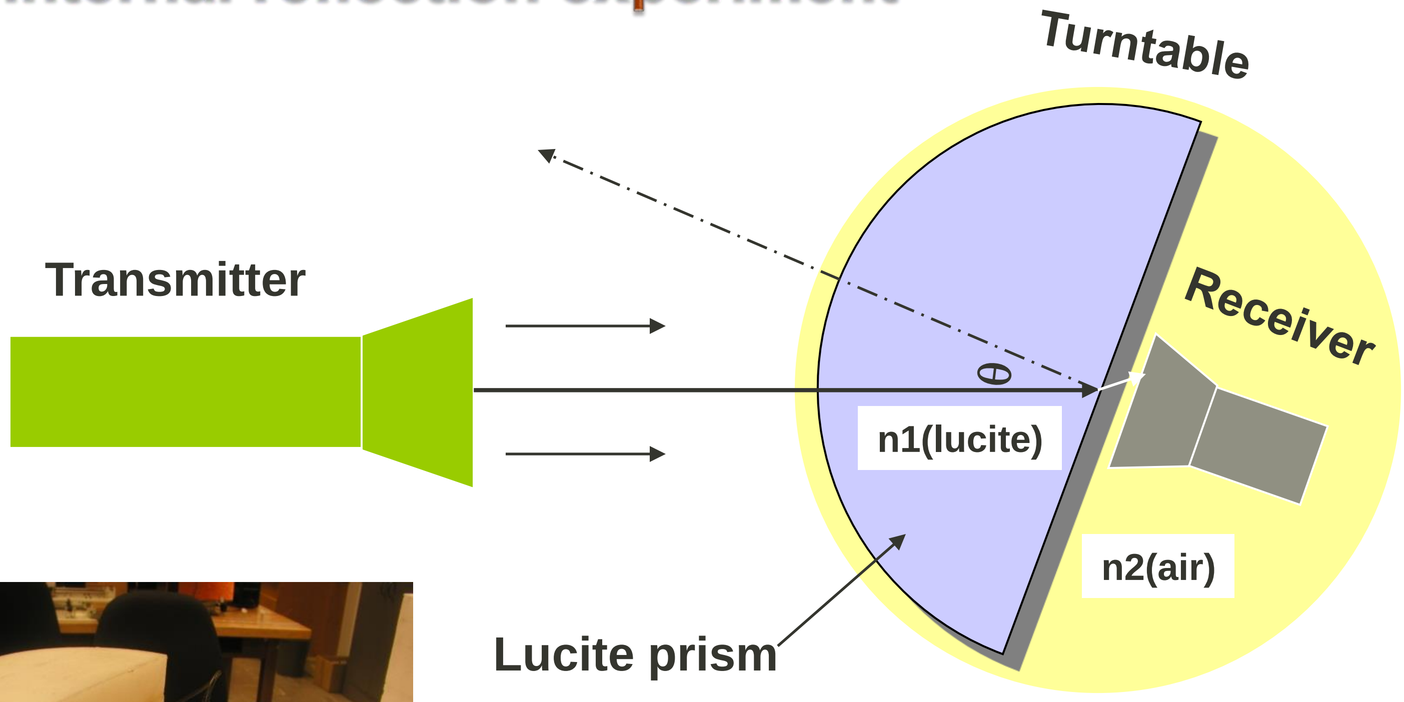
Claudius Ptolemaeus
after AD 83–*c.*168)

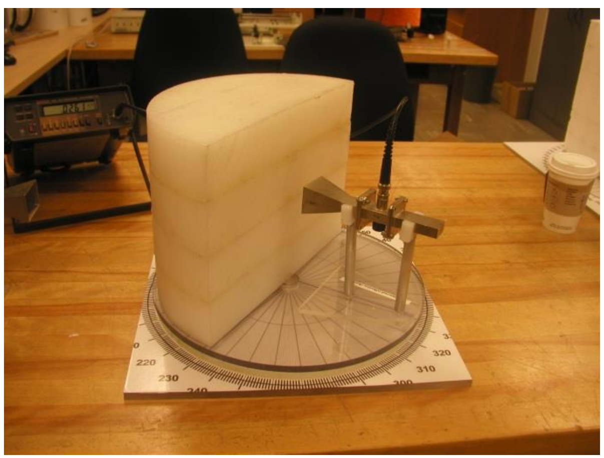
Equation for critical angle:

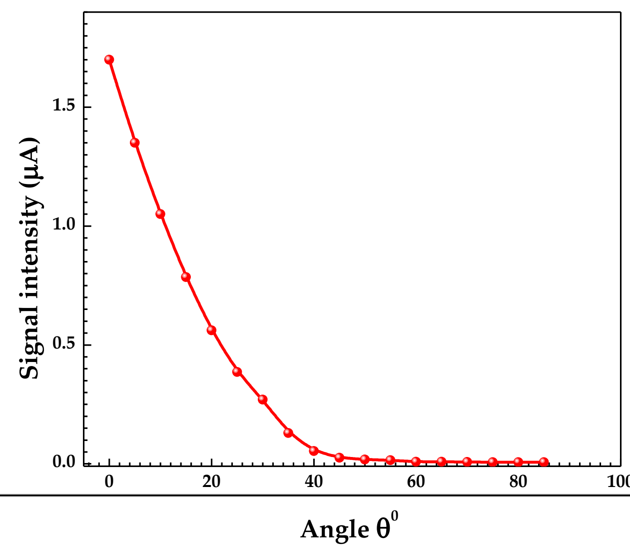
$$n_1 \sin\theta_c = n_2 \sin 90^o$$

$$\theta_c = \sin^{-1}(n_2/n_1)$$

Total internal reflection experiment

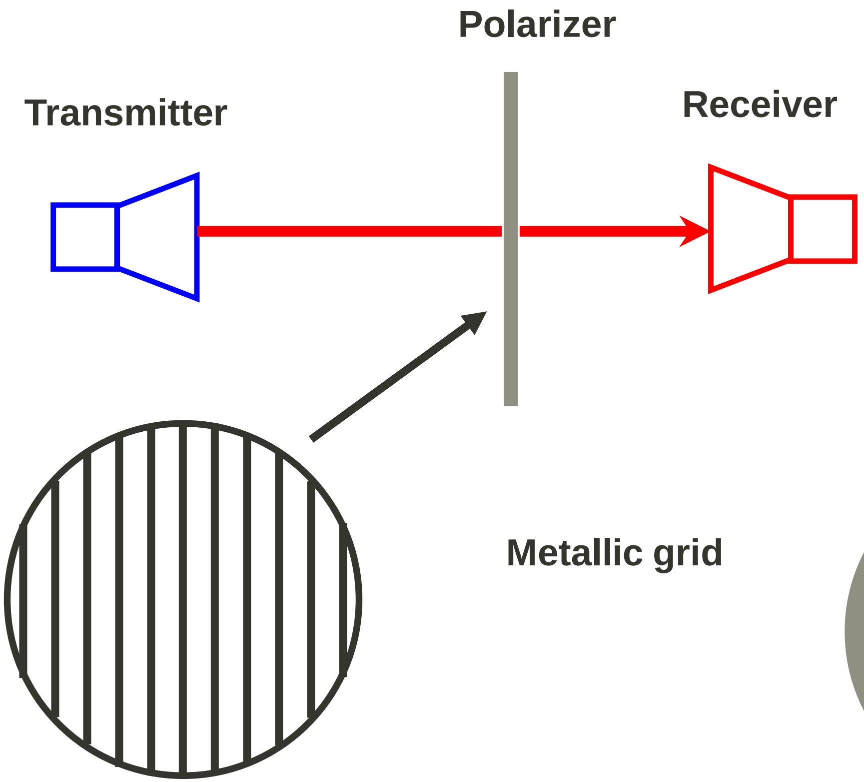


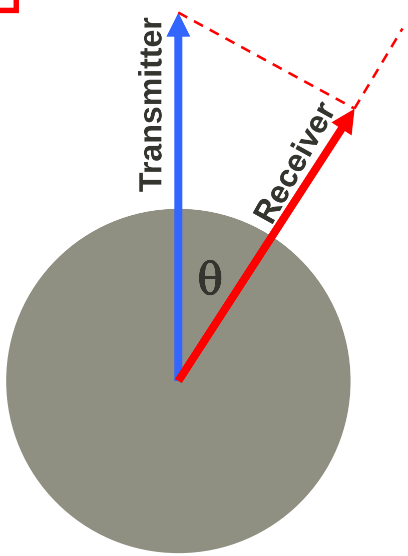




Experimental setup and the example of the data

Microwave polarization





Etienne-Louis Malus

1775 – 1812

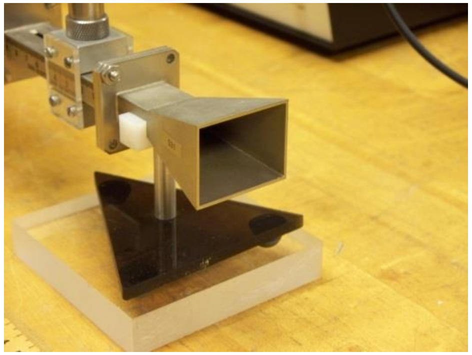
Malus law

$$E=E_0\cos\theta$$

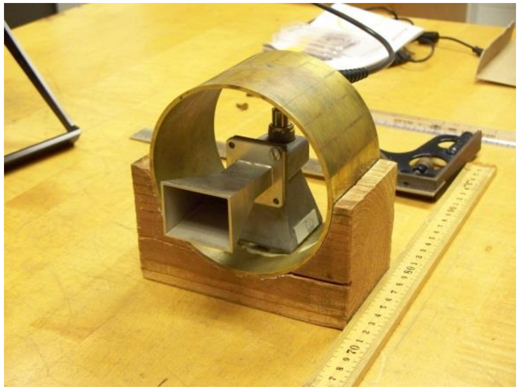
$$I \infty E^2$$

$$I=I_0\cos^2\theta$$

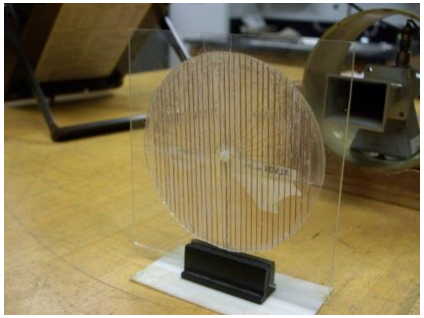
Microwave polarization



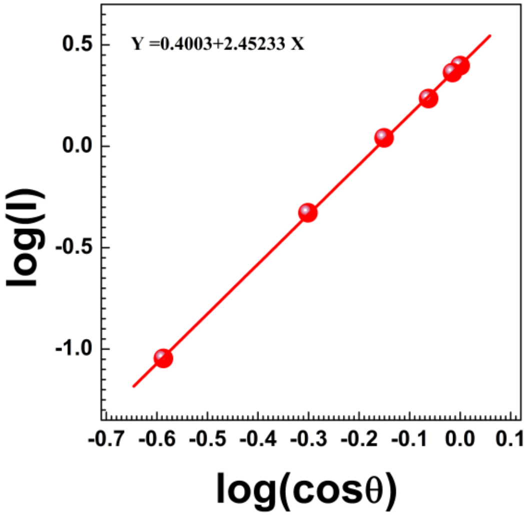
Transmitter



Rotatable receiver



Polarizer

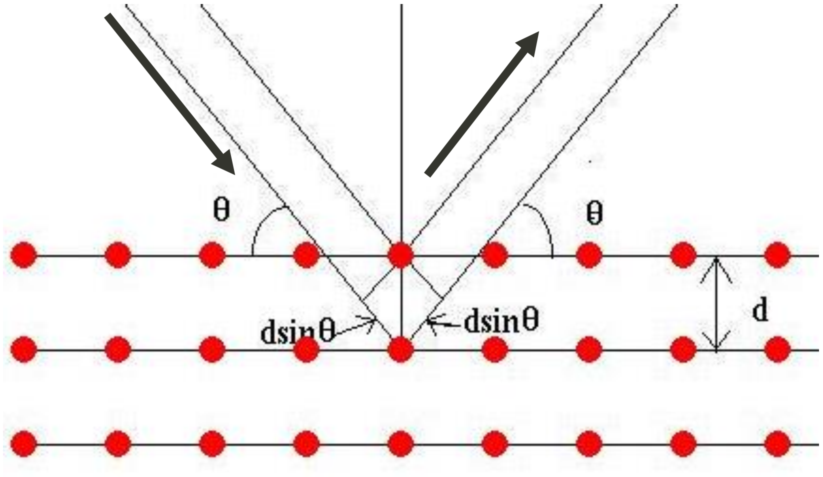


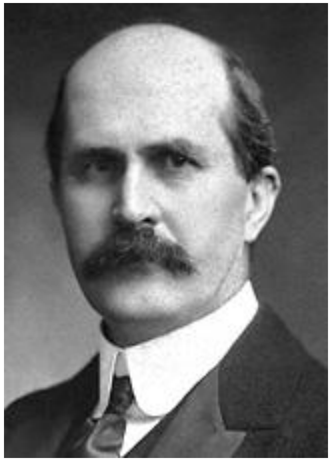
Experimental data

$I = I_0 \cos^2\theta$

Bragg diffraction

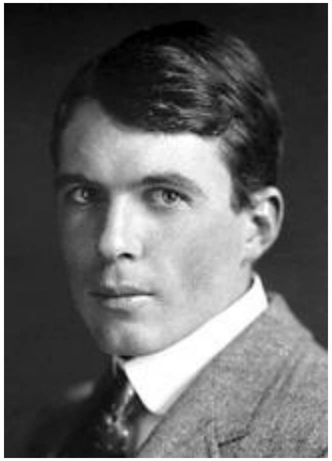
Interference of the EM waves reflected from the crystalline layers





Sir William Henry Bragg

1862-1942

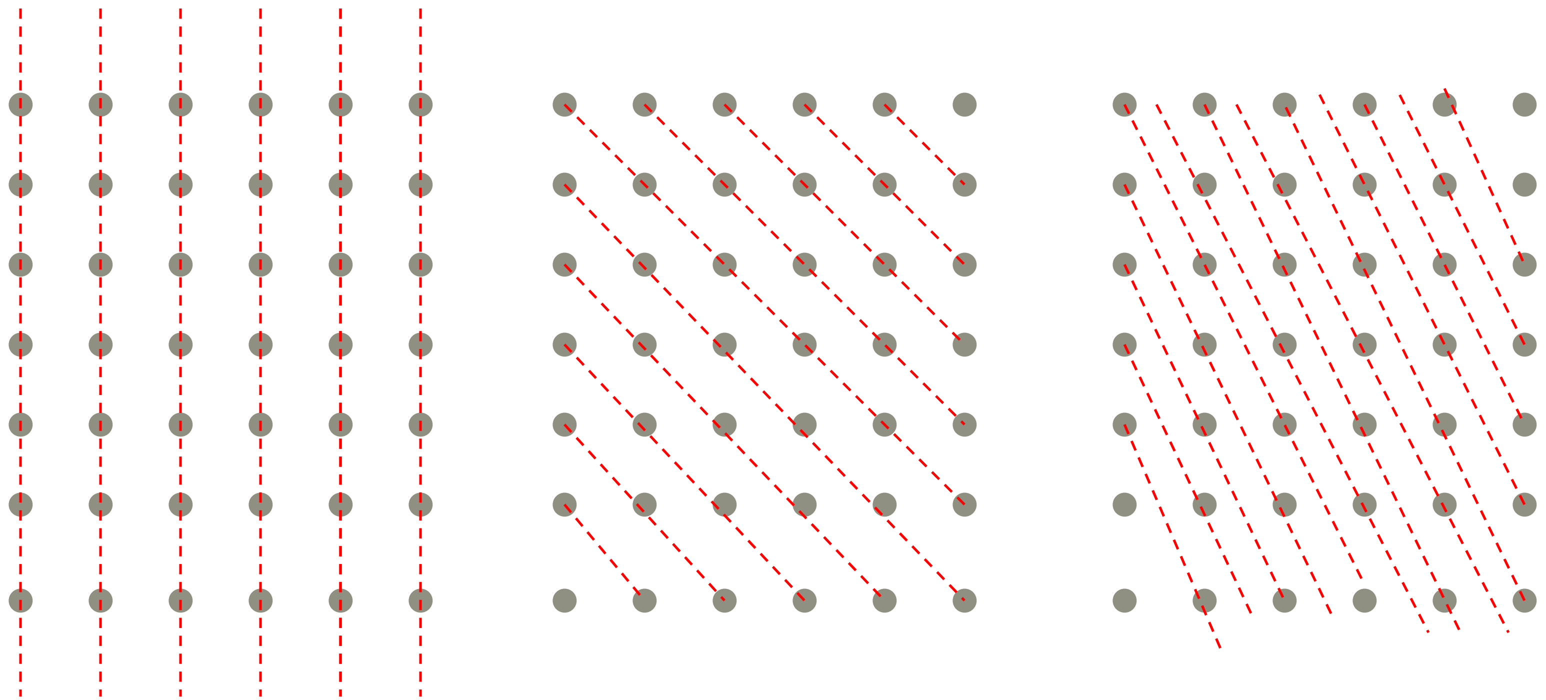


William Lawrence Bragg
1890-1971

The Nobel Prize in Physics 1915
"for their services in the analysis of crystal structure by means of X-rays"

$$n\lambda = 2d\sin\theta$$ *Bragg's Law*

Bragg diffraction



(100) (110) (210)

Different orientations of the crystal

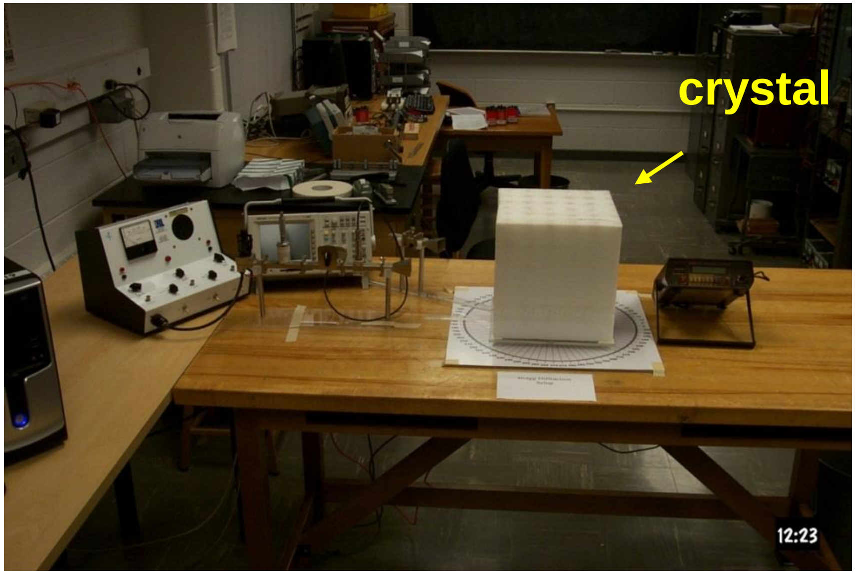
Bragg diffraction

In our experiment λ~3cm;
For cubic symmetry the angles of Bragg peaks can be calculated from:

$n\lambda = 2d\sin\theta$

$\lambda < 2d$

$$\left(\frac{\lambda}{2d}\right)^2 = \frac{\sin^2\theta}{h^2 + k^2 + l^2}$$

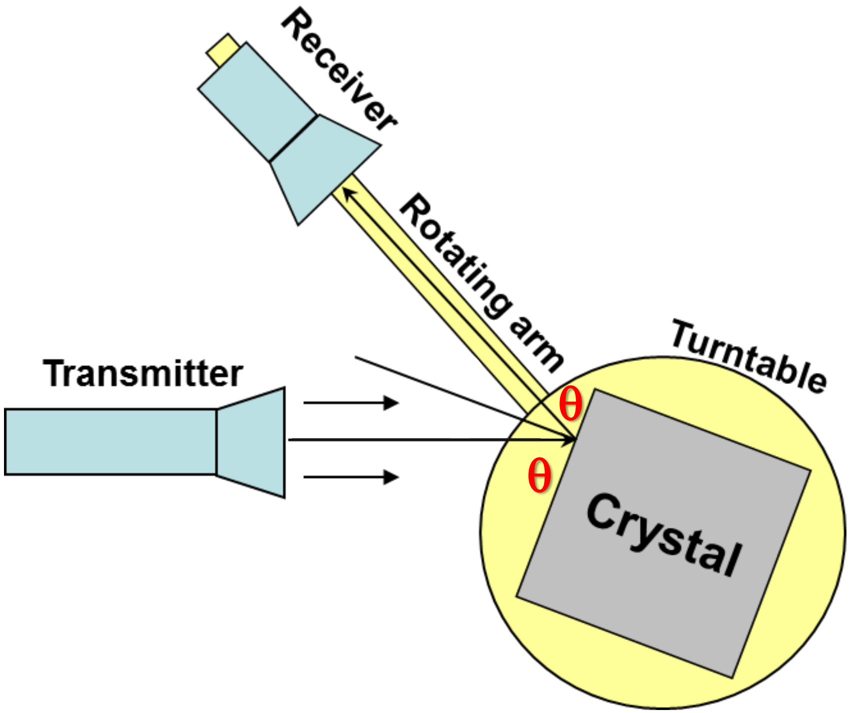


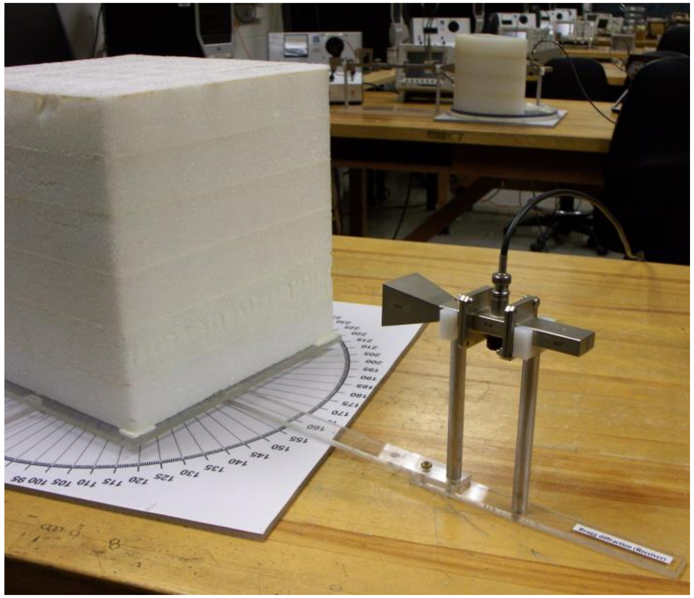
Experimental setup

where h,k,l are the Miller Indices.

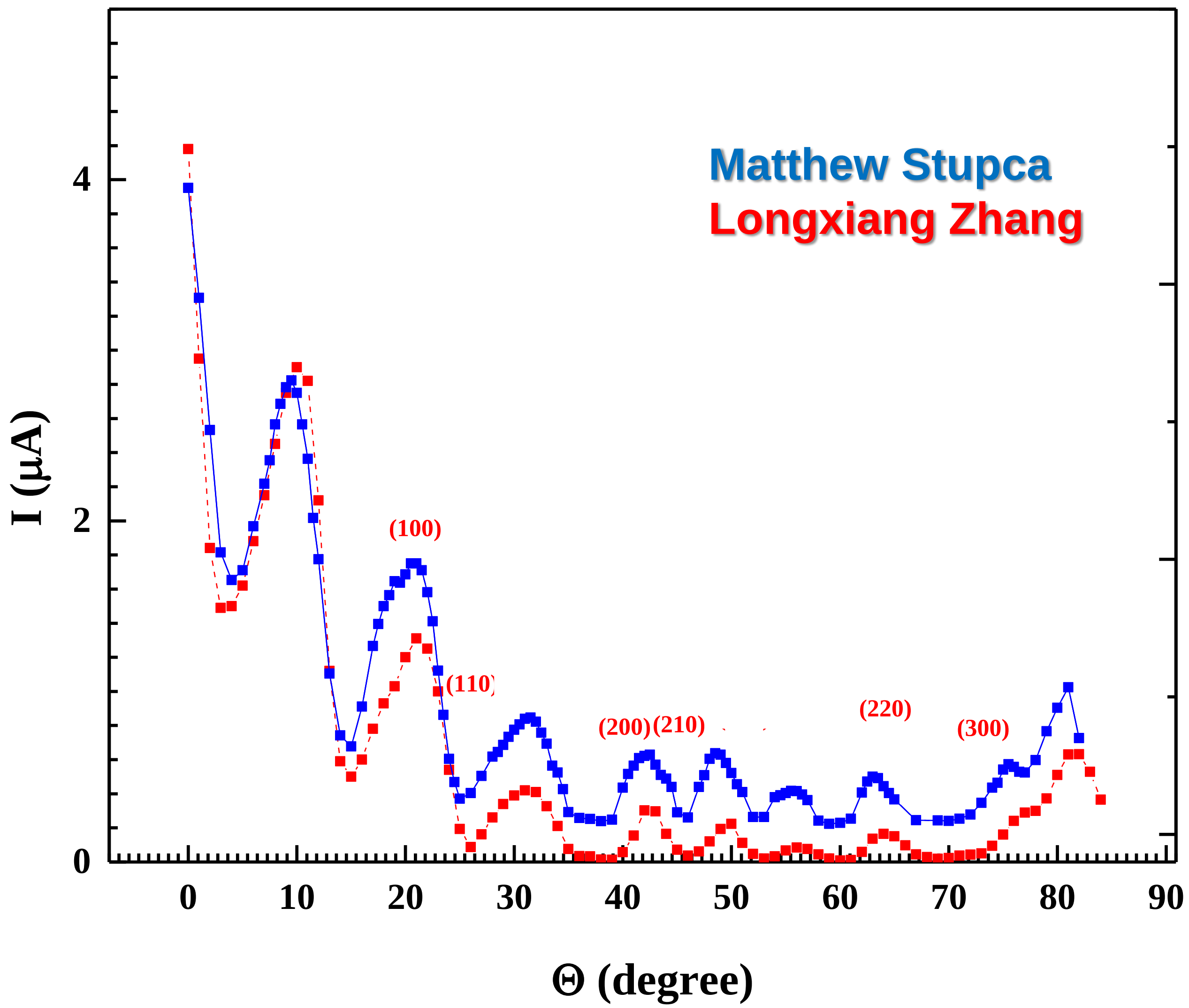
For crystal with d=5cm and λ=3cm the 3 first Bragg peaks for (100) orientation can be found at angles: ~17.5°; 36.9° and 64.2°

Bragg diffraction



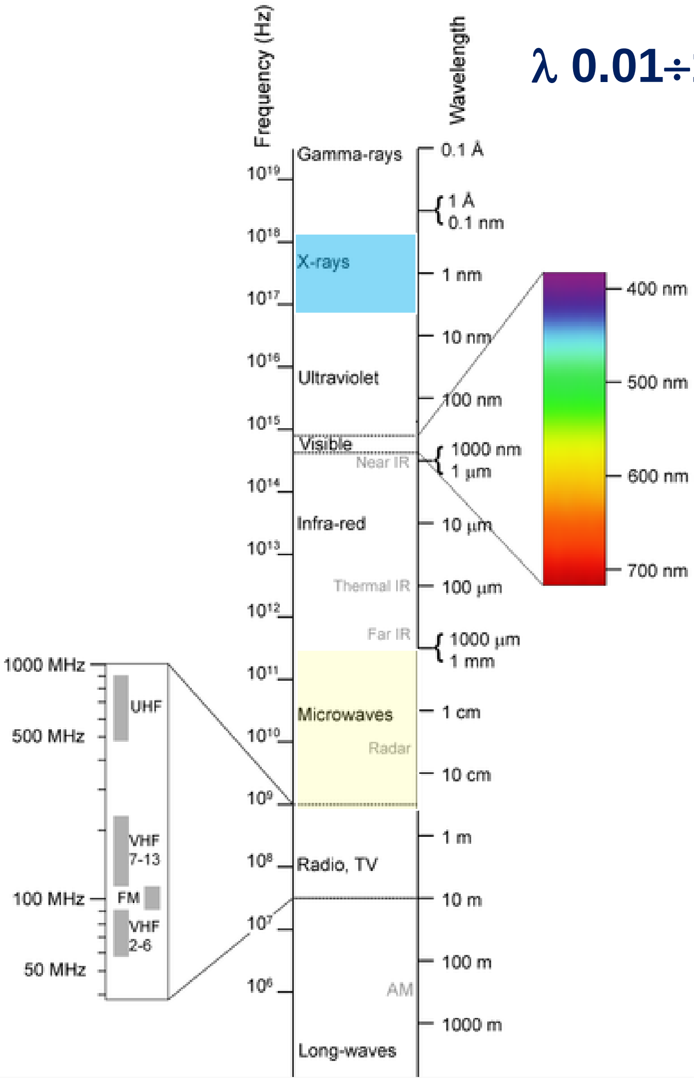


Bragg diffraction. Results.*



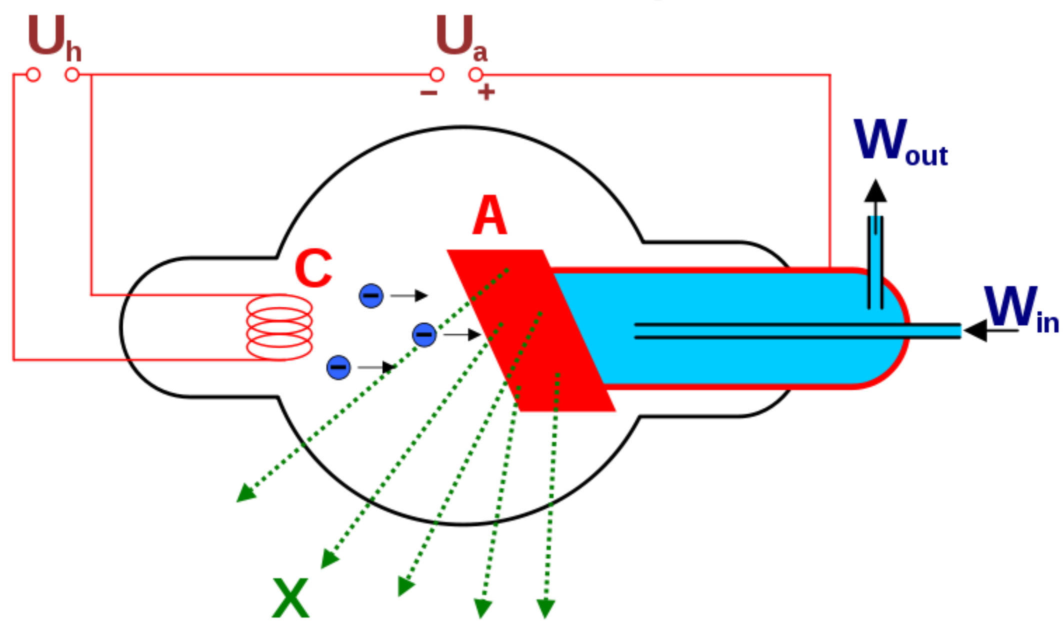
*courtesy of Matthew Stupca

Bragg diffraction. X-rays.



λ 0.01÷10nm

X-ray tube



*courtesy of Wikipedia

Bragg diffraction. X-rays.

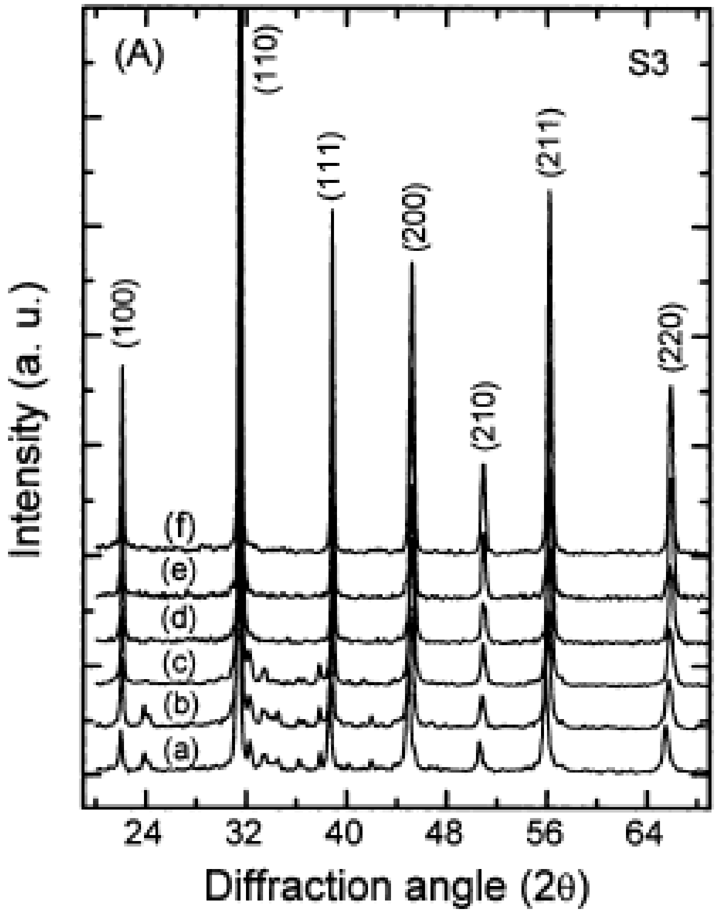
X-ray K-series spectral line wavelengths (nm) for some common target materials

Target	$K\beta_1$	$K\beta_2$	$K\alpha_1$	$K\alpha_2$
Fe	0.17566	0.17442	0.193604	0.193998
Co	0.162079	0.160891	0.178897	0.179285
Ni	0.15001	0.14886	0.165791	0.166175
Cu	0.139222	0.138109	0.154056	0.154439
Zr	0.70173	0.68993	0.78593	0.79015
Mo	0.63229	0.62099	0.70930	0.71359

David R. Lide, ed. (1994). *CRC Handbook of Chemistry and Physics 75th edition*. CRC Press. pp. 10–227

***courtesy of Matthew Stupca**

Bragg diffraction. X-rays.



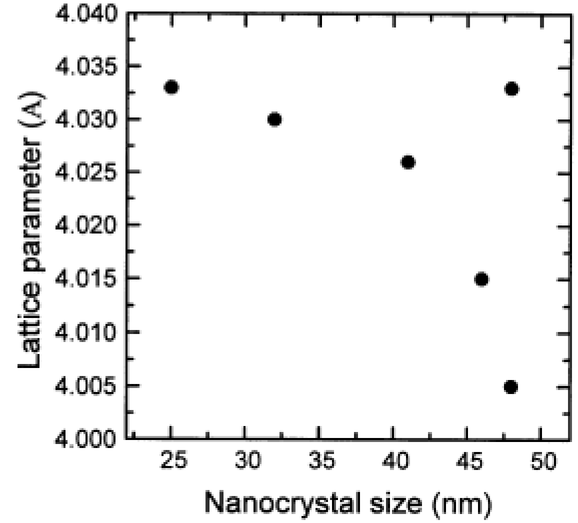


Fig. 4. Lattice parameter c versus the grain size in the $BaTiO_3$ nanocrystal.

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Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang[a,*], Zhen Yin[a], Qiang Chen[a], Weifeng Zhang[b], Wanchun Chen[c]

***courtesy of Matthew Stupca**

Comments and suggestions

1. Klystron is very hot and the high voltage (~300V) is applied to repeller.
2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.
3. Do not put on the tables any extra stuff – this will cause extra reflections of microwaves and could result in smearing of the data.
4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week

Good luck !